Bicocca-FT-03-32
UND-HEP-03-BIG 07
hep-ph/0312001

A 'BPS expansion' for B and D mesons

Nikolai Uraltsev*

Department of Physics, University of Notre Dame du Lac, Notre Dame, IN 46556, USA
and
INFN, Sezione di Milano, Milan, Italy

Abstract

We analyze consequences of the approximation $\mu_\pi^2 - \mu_G^2 \ll \mu_\pi^2$ (a 'BPS' limit) for B and D mesons. It is shown that neglecting perturbative effects many power corrections would vanish to all orders in $1/m_Q$, in particular those violating heavy flavor symmetry. Among them are corrections to $B \to D$ formfactors. A number of relations receive corrections only to the second order expanding around the BPS limit to any order in $1/m_Q$, including both f_+ and f_- at zero recoil. This allows an accurate evaluation of $\mathcal{F}_+$ for $B \to D\,\ell\nu$. Its perturbative renormalization is computed analytically in the required Wilsonian scheme, yielding the dominant 3% enhancement.

*On leave of absence from St. Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg 188300, Russia

The heavy quark expansion is a powerful tool for treating strong interactions relying on the first principles of QCD. Its indispensable part is a general Wilsonian separation of effects originating at sufficiently small and larger distances, a general concept of the operator product expansion (OPE) [1, 2]. The resulting expansion is most informative for inclusive decay probabilities where the OPE in local heavy quark operators emerges [3, 4]. Comprehensive application of all elements of the heavy quark theory often allows to put nontrivial constraints on the nonperturbative parameters in individual transition amplitudes based on information from inclusive decays.

An important role here is played by the heavy quark sum rules, including recently suggested spin sum rules [5] for the Small Velocity (SV) regime [6]. For example, they put a rigorous bound $\varrho^2 \geq \frac{3}{4}$ on the slope of the IW function and lead to a number of other inequalities in the heavy quark limit. The recently proposed generalizations to higher orders in velocity [7] resulted in remarkable relations (D'Orsay sum rules) and nonperturbative bounds for its higher derivatives. The constraining power of the whole set of the SV sum rules depends strongly on the actual size of the B meson expectation value μ_π^2 of the leading local heavy quark nonperturbative kinetic operator $\bar{Q}(i\vec{D})^2Q$. Its theoretical expectations used to be a controversial subject for years. The inequality between the expectation values of properly defined kinetic and chromomagnetic operators puts a nontrivial lower bound on μ_π^2.

Although strong dynamics at small and large distances is governed by the same QCD equations of motion, physics originating from below and above a GeV scale is quite different. It has been pointed out [8] that experiment may implicate an interesting pattern for the nonperturbative domain in B and D mesons, by favoring (see, e.g. Ref. [9]) μ_π^2 in the lower part of the allowed domain,[1] only little exceeding chromomagnetic expectation value $\mu_G^2(1\,\mathrm{GeV}) = 0.35^{+.03}_{-.02}\,\mathrm{GeV}^2$. In this case it is advantageous to analyze nonperturbative dynamics combining the heavy quark expansion with expanding around the point where $\mu_\pi^2 = \mu_G^2$ would hold.

This is not just an arbitrary point of a continuum in the parameter space, but a quite special limit where the heavy flavor ground state has to satisfy functional relation $\vec{\sigma}\vec{\pi}|B\rangle = 0$, with σ and π being heavy quark spin and momentum, respectively. From this perspective it is reminiscent to the 'BPS'-saturated state like the lowest Landau level for an electron in magnetic field.[2] It is not clear how deep the analogy with BPS symmetry goes, for the Pauli Hamiltonian $\frac{(\vec{\sigma}\vec{\pi})^2}{2m_Q}$ describes the leading-order power corrections rather than the static heavy quark Hamiltonian itself

$$\mathcal{H}_\infty = \mathcal{H}_{\mathrm{light}} - \int \mathrm{d}^3\mathbf{x}\, Q^+ A_0 Q\,(\mathbf{x}) \tag{1}$$

shaping the ground state. The functional relation would not be generally respected by short-distance perturbative exchanges. It can only be viewed as an approximate property

[1] Certain theoretical loopholes in the framework applied in individual analyses have been identified [10, 11, 12] which are expected to be eliminated in the forthcoming experimental results.

[2] The analogy to this problem in quantum mechanics and possible relation to BPS symmetry were first noted by M. Voloshin (1999, private communication), whom the suggested name of the limit ascends to.

of nonperturbative dynamics at energies below or about 1 GeV scale, and applied to the ground state.

A practical consequence of the proximity to the BPS regime is small room the heavy quark sum rules leave for possible values of the IW slope

$$\frac{3}{4} \lesssim \varrho^2(1\,\mathrm{GeV}) \lesssim 0.95 \quad \text{at } \mu_\pi^2(1\,\mathrm{GeV}) \lesssim 0.45\,\mathrm{GeV}^2 \,. \tag{2}$$

A later UKQCD lattice evaluation [13] $\varrho^2 = 0.83^{+.15+.24}_{-.11-.01}$ fit well that prediction, although precision remains insufficient. This and some other consequences were noted in Ref. [8] (see also [10, 12]). In the present Letter we examine this remarkable limit in more detail. We will largely abstract from the perturbative corrections, in particular those renormalizing power corrections to the heavy quark Lagrangian. To elucidate underlying physics we often use quantum-mechanical language; correspondence with second-quantized notations in field theory is given, for instance in Ref. [14]. The necessary introduction into heavy quark expansion technique and used notations can be found there, as well as in reviews [15, 16]. One of the practical applications of the 'BPS expansion' is an accurate model-independent determination of $|V_{cb}|$ from the $B \to D\,\ell\nu$ rate near zero recoil if enough statistics is available for this mode. Decays $B \to D\,\tau\nu_\tau$ sensitive to possible Higgs effects add motivation for a precision control. Power corrections to the decay amplitudes in this kinematics are obtained through order $1/m_Q^2$.

1 'BPS' relations for B and D mesons

The starting consequence of the BPS limit is equality of the spin-singlet and spin-nonsinglet expectation values appearing in the SV sum rules for asymptotically heavy quarks:

$$\varrho^2 = \frac{3}{4}, \qquad \overline{\Lambda} = 2\overline{\Sigma}, \qquad \rho_{LS}^3 = -\rho_D^3 \,. \tag{3}$$

Similar relations hold for nonlocal correlators of the $1/m_Q$ terms in the heavy quark Lagrangian:

$$\rho_{\pi G}^3 = -2\rho_{\pi\pi}^3, \qquad \rho_A^3 + \rho_{\pi G}^3 = -(\rho_{\pi\pi}^3 + \rho_S^3), \tag{4}$$

a series extending to higher-order correlators.

The BPS limit actually generalizes the heavy flavor symmetry to all orders in $1/m_Q$. First, all terms $\propto 1/m_Q^k$ in the $1/m_Q$ expansion of the effective Hamiltonian annihilate the ground state. Then all corresponding T-products likewise vanish in it, and no power corrections to the heavy quark relation $M_P = m_Q + \overline{\Lambda}$ appear, so that

$$M_B - M_D = m_b - m_c \tag{5}$$

holds to all order in $1/m_Q$. It is also important that the Foldy-Wouthuysen transformation acts trivially (is unity) on the ground state, therefore the proper nonrelativistic wavefunction coincides with the upper component of the full Dirac bispinor in the meson restframe.

The above facts follow from the observation that for the 'BPS' wavefunction the lower component of the bispinor does not appear even when power correction are included. To prove this, we recall the full QCD equation of motion for the quark field in the nonrelativistic notations. If the static solution φ_0 for $m_Q \to \infty$, $\pi_0\varphi_0 = 0$ additionally satisfies the 'BPS' constraint $(\vec\sigma\vec\pi)\varphi_0 = 0$, the bispinor

$$Q(x,t) = e^{-im_Q t} \begin{pmatrix} \varphi_0(x,t) \\ 0 \end{pmatrix} \tag{6}$$

solves the Dirac equation

$$(\not\pi + m_Q\gamma_0)Q = m_Q\, Q\;, \qquad \pi_\mu \equiv iD_\mu - m_Q v_\mu\;, \tag{7}$$

and the corresponding wavefunction

$$\Psi_\alpha(\vec x_Q, \{x_{\rm light}\}) \;=\; \begin{pmatrix} \Psi^0_\alpha(\vec x_Q, \{x_{\rm light}\}) & \alpha\!=\!1,2 \\ 0 & \alpha\!=\!3,4 \end{pmatrix} \tag{8}$$

is a formal eigenstate with $E = m_Q + \overline\Lambda$ of the finite-m_Q Hamiltonian including light degrees of freedom – their wavefunction simply does not depend on m_Q.

Absence of nontrivial Foldy-Wouthuysen corrections in the ground state is also transparent. Since time derivative in the QCD Lagrangian for fermions enters linearly with the gauge-field–independent coefficient, the only source of the transformation to nonrelativistic wavefunction is eliminating the lower component of the bispinor. If the latter does not appear, Eq. (6), no transformation is required. This corresponds to the fact that Foldy-Wouthuysen transformation $S_{\rm FW}$ (considered in the space of the upper components, in the restframe) can be viewed as

$$S_{\rm FW} = \left(\frac{1+\gamma_0}{2}\right)^{-\frac{1}{2}} = 1 + \frac{(\vec\sigma\vec\pi)^2}{8m_Q^2} + \frac{-\frac{1}{2}(\vec D\vec E)+\frac{1}{2}\vec\sigma\cdot\{\vec E\times\vec\pi - \vec\pi\times\vec E\}}{8m_Q^3} + ... \tag{9}$$

where the inverse square root of the projection operator is understood as the local OPE expansion in $1/m_Q$ of the forward matrix element $\bar Q\left(\frac{1+\gamma_0}{2}\right)^{-\frac{1}{2}} Q$ in terms of the upper components. As shown in Ref. [17], this expansion generates the whole nonrelativistic expansion for Dirac Hamiltonian, so triviality of Foldy-Wouthuysen transformation is intimately related to absence of corrections to the meson mass.

Vanishing of corrections to heavy flavor symmetry in $1/m_Q$ expansion applies also to the transition amplitudes between the ground-state mesons with different heavy quarks, like in $B \to D$ decays. Below we take a closer look at the $B \to D$ amplitude assuming both b and c are heavy enough to meaningfully apply the expansion. Since axial current does not contribute here, we focus on $\bar c\gamma_\mu b$-induced amplitudes; the results apply to other allowed Lorentz structures as well (say, scalar relevant for charge Higgs contributions).

In general, the $B \to D$ amplitude is described by two vector formfactors

$$\langle D(p_2)|\bar c\gamma_\nu b|B(p_1)\rangle = f_+(\vec q^{\,2})(p_1\!+\!p_2)_\nu + f_-(\vec q^{\,2})(p_1\!-\!p_2)_\nu\;. \tag{10}$$

At zero recoil, $\vec{q}=0$ a single amplitude, viz. $\nu=0$ remains: $J_0=(M_B+M_D)f_+(0)+(M_B-M_D)f_-(0)$. In the heavy quark limit one has

$$f_+(0)=\frac{M_B+M_D}{2\sqrt{M_BM_D}}, \qquad f_-(0)=-\frac{M_B-M_D}{2\sqrt{M_BM_D}}\ . \tag{11}$$

It has been noted [10] that in the BPS limit all power corrections to J_0 vanish. A stronger statement applies: the f_+ formfactor determining all decays amplitudes with massless leptons, at zero recoil keeps its asymptotic heavy quark limit value

$$f_+(0)=\frac{M_B+M_D}{2\sqrt{M_BM_D}}\ . \tag{12}$$

Moreover, at arbitrary momentum transfer the heavy quark relation between f_+ and f_- remains valid in higher orders in $1/m_Q$:

$$f_-(q^2)=-\frac{M_B-M_D}{M_B+M_D}f_+(q^2)\ . \tag{13}$$

Furthermore, dynamic power corrections in $B\to D$ amplitude vanish:

$$f_+(q^2)=\frac{M_B+M_D}{2\sqrt{M_BM_D}}\ \xi\left(\tfrac{M_B^2+M_D^2-q^2}{2M_BM_D}\right)\ , \tag{14}$$

where $\xi(v_\mu^D v_\mu^B)$ is the normalized formfactor in the infinite mass limit (IW function). Therefore, the $B\to D$ differential decay rate would more or less directly measure the IW function. Below we show a way to derive these BPS consequences.

To relate f_+ and f_- in Eq. (13) we can consider two independent amplitudes, $\bar{c}\gamma_0 b$ and $i\partial_\mu J_\mu$; applying the QCD equation of motion we have

$$i\partial_\mu J_\mu=(m_b-m_c)\,\bar{c}\,b\,, \qquad \langle D|i\partial_\mu J_\mu|B\rangle=(M_B^2-M_D^2)f_+ + q^2 f_-\ . \tag{15}$$

Both $\bar{c}\gamma_0 b$ and $\bar{c}b$ currents do not mix upper and lower components for bispinors. Therefore, if the initial B meson is at rest and its lower component vanishes, the two currents coincide up to the factor m_b-m_c leading to

$$(m_b-m_c)\,[(M_B+E_D)f_+ + (M_B-E_D)f_-]=(M_B^2-M_D^2)f_+ + (M_B^2+M_D^2-2M_BE_D)f_-\ . \tag{16}$$

Replacing m_b-m_c by M_B-M_D we arrive at the stated relation. Hence, it is a consequence of vanishing lower components and of absent power corrections to the meson masses in the BPS limit.

Non-renormalization of the (exact) zero-recoil amplitude can be readily seen, for instance, in the heavy quark sum rule for J_0 extended to arbitrary order in $1/m_Q$: the r.h.s. of the sum rule would not get corrections,

$$|F_D|^2+\sum_{\rm excit}|F_i|^2=1-\tfrac{\mu_\pi^2-\mu_G^2}{4}\left(\frac{1}{m_c}-\frac{1}{m_b}\right)^2-\tfrac{\rho_D^3+\rho_{LS}^3}{4}\left(\frac{1}{m_c}+\frac{1}{m_b}\right)\left(\frac{1}{m_c}-\frac{1}{m_b}\right)^2-\ ...\ , \tag{17}$$

and inelastic transition amplitudes likewise vanish order by order in the BPS limit. The simplest way to understand this, however is using its quantum mechanical meaning revealed in Ref. [18].[3] We have seen that no corrections to the quantum mechanical wavefunction appear, lower components are absent and there is no explicit corrections in the $\bar{c}\gamma_0 b$ current – then no room for power corrections remains.

Absence of corrections to the heavy quark limit relation (14) can be understood in the following way, looking once again at the timelike component of the current. Assuming B at rest and D moving with fixed velocity, we observe that the amplitude does not depend on m_b – lower component of the b field is absent, while the upper one remains mass-independent. Therefore, it can only be a function of m_c and can be computed at $m_b \to \infty$. Changing the roles of b and c we would find it rather can only be a function of m_b. Therefore it has to be free from any power corrections, at a given mesons velocity.

Thus, the heavy flavor symmetry for the ground state would extend to any order in $1/m_Q$ expansion in the strict BPS limit.

What if $m_Q \to 0$?

Absence of all power corrections to heavy quark relations even in a very special regime may sound paradoxical. We definitely know that when $m_Q \to 0$ usual B and D mesons would rather evolve to become counterparts of the Goldstone particles like π and K. The mass relation for them would be clearly different, for instance.

There is no contradiction between the two regimes even if we abstract from the fact that the BPS limit cannot be exact and the corrections to it analyzed in the $1/m_Q$ expansion would explode when m_Q descends below a certain hadronic mass. The scale where the BPS-protected heavy quark relations get completely destroyed may even not decrease but remain stable when approaching the true BPS limit. For power expansion cannot have a finite radius of convergence being only asymptotic. Even if all power terms would vanish, there remain exponential terms scaling like

$$e^{-\frac{2m_Q}{\mu_{\rm hadr}}} \tag{18}$$

which modify the heavy quark relations.

Presence of such effects is a general feature of expansions in field theory. In the heavy quark expansion they can explicitly appear due to unlimited spectral density of the operator π_0 (heavy quark Hamiltonian), with the support stretching below $-2m_Q$. Heavy quark expansion excludes extra heavy quark degrees of freedom expanding $1/(2m_Q+\pi_0)$ in $1/2m_Q$. The expansion is convergent only if $|\pi_0| < 2m_Q$ in the operator sense, which seems impossible for a state well localized in space. Therefore, we should rather expect that violation of the BPS relations becomes of order unity below a certain hadronic scale regardless of proximity to the BPS regime for sufficiently heavy quarks.

There are transparent mechanisms for such a change in the regime. An obvious candidate is spontaneous breaking of the chiral symmetry through nontrivial condensate $\langle \bar{Q}Q \rangle$

[3] The $1/m_Q^3$ term for the sum rule for $B \to D^*$ was derived in Ref. [15]. The simplest way, in fact uses this QM interpretation and explicit form of Foldy-Wouthuysen transformation, Eq. (9).

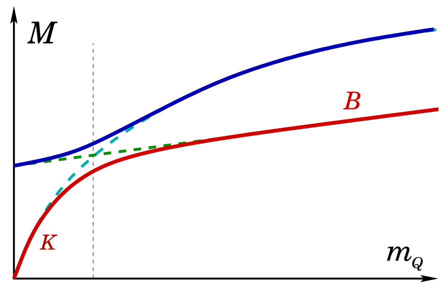

Figure 1: 'Level crossing' would completely change m_Q-behavior below a certain hadronic mass even at arbitrary weak "BPS perturbation"

of the $Q\bar{Q}$-pairs in the physical vacuum when the quark becomes light. The basic assumption behind heavy quark symmetry, in contrast is separate conservation of numbers of Q and $\bar{Q}$. Even if a formal solution of equations of motion exists, it may not be the correct ground state over the true vacuum.

Another related mechanism is usual quantum mechanical level crossing. If analytically continued eigenvalues for two states happen to cross at some value of the heavy quark mass, the "repulsion" occurs, and behavior of the true ground state below and above the crossing mass becomes totally different, while the effect of any perturbation mixing the states becomes of order unity regardless of its formal strength, see Fig. 1.

2 Expanding around the BPS regime

The BPS limit cannot be exact in actual QCD, rather an approximate property of strong dynamics in the nonperturbative domain of momenta below a GeV scale. Therefore, like with the conventional $1/m_Q$ expansion it is important to determine the scale of violation of its particular predictions. If a concrete low-order in $1/m_Q$ correction is found in terms of the heavy quark operators, it is not difficult to see its BPS scaling. We, however, need a deeper classification which would hold to all orders in $1/m_Q$.

The practical expansion parameter to quantify deviations from the BPS limit, the norm of the state obtained by acting $\vec{\sigma}\vec{\pi}$ on the ground state

$$\| \, \vec{\sigma}\vec{\pi} \, |B\rangle \| = \sqrt{\mu_\pi^2 - \mu_G^2} \, , \tag{19}$$

has dimension of mass. A similar dimensionless parameter is

$$\beta = \|\pi_0^{-1}(\vec{\sigma}\vec{\pi}) \, |B\rangle\| \equiv \sqrt{3\Big(\varrho^2 - \tfrac{3}{4}\Big)} = 3\Big[\sum_n \, |\tau_{1/2}^{(n)}|^2\Big]^{\frac{1}{2}} \tag{20}$$

(π_0^{-1} in quantum mechanical notations is simply $-\frac{1}{\mathcal{H}_\infty - \overline{\Lambda}}$ here). Numerically β is not too small, similar in size to generic $1/m_c$ expansion parameter in conventional $1/m_c$ series. Relations violated to order β may in practice be more of a qualitative nature, while

$\beta^2 \propto \frac{\mu_\pi^2-\mu_G^2}{\mu_\pi^2}$ can provide enough suppression. Moreover, we can count together powers of $1/m_c$ and powers of β to judge the real quality of a particular heavy quark relation.

BPS relations (13), (14) for the decay amplitude at arbitrary recoil indeed receive corrections $\propto \beta^1$, likewise equality of $\rho_{\pi G}^3$ and $-2\rho_{\pi\pi}^3$. Other relations between the heavy quark parameters mentioned in Sect. 1 are accurate up to terms β^2. In particular,

- $M_B - M_D = m_b - m_c$ and $M_D = m_c + \overline{\Lambda}$.
- Zero recoil matrix element $\langle D|\bar{c}\gamma_0 b|B\rangle$ is unity up to β^2.
- Experimentally measured $B \to D$ formfactor f_+ near zero recoil receives only second-order corrections in β to all orders in $1/m_Q$:

$$f_+(0) = \frac{M_B + M_D}{2\sqrt{M_B M_D}} + \mathcal{O}(\beta^2) \; . \tag{21}$$

This is an analogue of the Ademollo-Gatto theorem for the BPS expansion. The similar statement clearly applies to f_- as well.

Absence of β^1 corrections to the meson mass is easily seen in usual perturbation theory:

$$M_B = \langle B|\mathcal{H}_{\rm tot}|B\rangle \; . \tag{22}$$

The terms linear in β would reside either in $|B\rangle$ or in $\mathcal{H}_{\rm tot}$. The former vanishes since to the leading, β^0 order $|B\rangle$ remains the same BPS state at finite m_b. Absence of the linear in β terms from $\mathcal{H}_{\rm tot}$ is also understood – all local operators there include left-most $\vec{\sigma}\vec{\pi}$ acting on $\bar{b}$ and right-most $\vec{\sigma}\vec{\pi}$ acting on b since originate from excluding the lower bispinor components; this is transparent in the approach described in Ref. [17].[4]

Absence of linear non-BPS effects in zero-recoil $\langle D|\bar{c}\gamma_0 b|B\rangle$ is also transparent in BPS perturbation theory. Corrections $\propto \beta$ can originate either from charm or from beauty wavefunction, but not from both simultaneously,

$$\delta_{\beta^1}\langle D|\bar{c}\gamma_0 b|B\rangle = \langle \delta_{\beta^1}\Psi_D|\Psi_B^0\rangle + \langle \Psi_D^0|\delta_{\beta^1}\Psi_B\rangle = 0 \; , \tag{23}$$

where we have used that the current acts on the unperturbed BPS state as a unit operator except changing flavor. Alternatively, this follows from the zero-recoil vector sum rule as described in the preceding section – power corrections to the sum rule appear only to order β^2, and excitation transition amplitudes appear to order β.

Vanishing of $\mathcal{O}(\beta)$ corrections in Eq. (21) for $f_+(0)$ is least obvious. Since this holds for the combination of f_+ and f_- describing J_0, it suffices to show this for an alternative combination, say taking the spacelike current $\bar{c}\vec{\gamma}b$ in the case where one of the mesons is at rest and retaining only linear in velocity terms. Once again we make use of the fact that order-β^1 effects could originate from perturbation in either B or D wavefunctions, but not both simultaneously. To simplify algebra we can assume that B meson is at rest if BPS corrections appear in B, and go to the D restframe if a deviation from the BPS state occurs in charm sector.

[4] In particular, this follows from its Eq. (10) once the scalar expectation value $\langle B|\bar{b}b|B\rangle$ is $1 + \mathcal{O}(\beta^2)$. The energy-momentum tensor $\mathcal{D}$ does not depend explicitly on m_b and plays the role of the rest-frame Hamiltonian of light degrees of freedom.

The next helpful observation is that if, say the beauty sector wavefunction enters to the leading order in β, it can be assumed to be in the more familiar $m_b\to\infty$ limit. Projecting the matrix element on $\vec{v}$ we then get the spinor structure

$$\vec{n}\vec{J} \;=\; 2\sqrt{M_B M_D}\,\begin{pmatrix}\varphi^{(c)}\\ \chi^{(c)}\end{pmatrix}^{\dagger}\begin{pmatrix}0 & \vec{\sigma}\vec{n}\\ \vec{\sigma}\vec{n} & 0\end{pmatrix}\begin{pmatrix}\varphi_0^{(b)}(\vec{v})\\ \frac{\vec{\sigma}\vec{v}}{2}\,\varphi_0^{(b)}\end{pmatrix}, \qquad \vec{n}=\frac{\vec{v}}{|\vec{v}\,|}\,, \tag{24}$$

where $\varphi^{(c)}$ and $\chi^{(c)}$ refer to D at rest and include all mass corrections.

The term $\langle\varphi^{(c)}|(\vec{\sigma}\vec{n})\frac{\vec{\sigma}\vec{v}}{2}|\varphi_0^{(b)}\rangle$ yields just the heavy quark limit result, Eq. (11) if $\varphi^{(c)}$ coincided with the asymptotic wavefunction $\varphi_0^{(c)}$. It generally does not, but the first-order corrections to the overlaps like $\langle\varphi_0^{(c)}|\varphi_0^{(b)}\rangle$ always vanish. This applies to BPS perturbations since $\varphi^{(c)}$ remains normalized to unity up to terms $\propto\beta^2$ to any order in $1/m_c$.

Another source of power corrections is the product $\langle\chi^{(c)}|(\vec{\sigma}\vec{n})|\varphi_0^{(b)}(\vec{v})\rangle$. Now we use that $\varphi_0^{(b)}(\vec{v})$ is the asymptotic heavy quark wavefunction which velocity dependence to the first order is given by

$$|\varphi_0^{(b)}(\vec{v})\rangle = |\varphi_0^{(b)}(0)\rangle - \frac{1}{\mathcal{H}_\infty-\overline{\Lambda}}\,\vec{\pi}\vec{v}\,|\varphi_0^{(b)}(0)\rangle \equiv |\varphi_0^{(b)}(0)\rangle + \pi_0^{-1}\,\vec{\pi}\vec{v}\,|\varphi_0^{(b)}(0)\rangle\,, \tag{25}$$

where $\mathcal{H}_\infty$ is the static heavy quark limit Hamiltonian, and the second form uses the analogue of the second-quantized notations. Since heavy quark spin $\vec{\sigma}$ commutes with $\mathcal{H}_\infty$ this piece reads

$$\tfrac{|\vec{v}|}{3}\,\langle D|\,\bar{c}\,\tfrac{1-\gamma_0}{2}\pi_0^{-1}(\vec{\sigma}\vec{\pi})\,b\,|B_0(\vec{v})\rangle \tag{26}$$

when calculated for spinless meson states. Therefore it vanishes for the BPS beauty state regardless of the charm wavefunction.

The above reasoning essentially relies on linear in $\vec{v}$ approximation. To order $\vec{v}^{\,2}$, for instance, the operator $\vec{\pi}^{\,2}$ appears along with $(\vec{\sigma}\vec{\pi})^2$, which does not vanish when acts on the BPS state. Therefore, while having managed to leave the point of zero recoil, we cannot extend the Ademollo-Gatto theorem to, say the slope of the formfactor.

3 Application to $B\to D\,\ell\nu$ near zero recoil

Since f_- formfactor does not contribute to any decay amplitude with massless leptons, the amplitude even near zero recoil depends on the space-like current which suffers from linear $1/m_c$ effects in the $1/m_Q$ expansion; conventionally this is viewed as the serious theoretical drawback of such a decay mode. The BPS expansion turns out more robust in this respect protecting the heavy quark relation up to the second order. This allows an accurate estimate of the $B\to D$ rate near zero recoil in terms of $|V_{cb}|^2$.

Similar to heavy quark symmetry itself, the BPS limit is affected by short-distance perturbative effects. Accounting for the latter should therefore be done accordingly. The Wilsonian procedure with the explicit 'hard' cutoff μ is most suitable here simply eliminating the low-momentum domain. The principal perturbative corrections are just short-distance renormalization of the bare $\bar{c}\gamma_\nu b$ current itself. The technique for such Wilsonian calculations has been elaborated and applied to zero-recoil $B\to D^*$ amplitude, see

Ref. [16]. The method specifically corresponds to the adopted renormalization scheme for nonperturbative operators with the upper cutoff μ on energy [8]. The case of $B\to D$ assuming non-vanishing velocity requires only technical modifications.

The one-loop result with arbitrary cutoff scale μ is obtained in an analytic form:

$$\begin{aligned}\xi_V(\mu)=1+\frac{2\alpha_s}{3\pi}\Bigg[&\frac{3m_b^2+2m_cm_b+3m_c^2}{2(m_b^2-m_c^2)}\ln\frac{\mu+\omega_b}{\mu+\omega_c}-2-\mu\Bigg\{\frac{4}{3\mu^2}\frac{m_c\omega_b-m_b\omega_c}{m_b-m_c}+\frac{2}{3}\frac{\frac{m_c}{\omega_b}-\frac{m_b}{\omega_c}}{m_b-m_c}\\ &-\frac{1}{3}\frac{\frac{\omega_b}{m_b}-\frac{\omega_c}{m_c}}{m_b-m_c}+\frac{2m_cm_b}{m_c+m_b}\frac{\frac{1}{\omega_b}-\frac{1}{\omega_c}}{m_b-m_c}+\frac{1}{6}\frac{1}{m_c+m_b}\left(\frac{\omega_c}{m_c}\left(3-\frac{m_b}{m_c}\right)+\frac{\omega_b}{m_b}\left(3-\frac{m_c}{m_b}\right)\right)\\ &+\frac{2}{3}\frac{2m_cm_b}{m_c+m_b}\left(\frac{m_b}{\omega_b^3}+\frac{m_c}{\omega_c^3}\right)+\frac{\mu}{6}\left(\frac{1}{m_c}-\frac{1}{m_b}\right)^2-\frac{2}{3}\frac{\mu^2}{m_cm_b}\frac{\frac{m_b^2}{\omega_c}+\frac{m_c^2}{\omega_b}}{m_b^2-m_c^2}\Bigg\}\Bigg]\;,\end{aligned}\tag{27}$$

where

$$\omega_c=\sqrt{m_c^2+\mu^2},\qquad \omega_b=\sqrt{m_b^2+\mu^2}\;. \tag{28}$$

Fig. 2 shows it numerically assuming $m_c=1.2\,\mathrm{GeV}$, $m_b=4.6\,\mathrm{GeV}$ and $\alpha_s=0.3$.

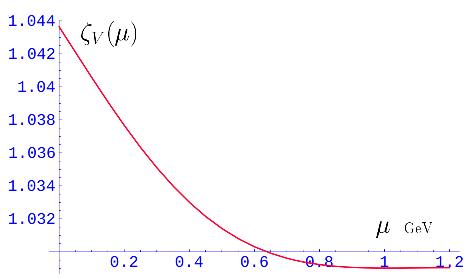

Figure 2: Short-distance renormalization for $B\to D$ amplitude at $m_c=1.2\,\mathrm{GeV}$, $m_b=4.6\,\mathrm{GeV}$ and $\alpha_s=0.3$ as a function of the separation scale μ.

The first-order $1/m_Q$ correction is readily read off the current in Eq. (24), cf. Eq. (26):[5]

$$\mathcal{F}_+=\frac{2\sqrt{M_BM_D}}{M_B+M_D}\,f_+(0)=\xi_V(\mu)+\left(\frac{\overline{\Lambda}}{2}-\overline{\Sigma}\right)\left(\frac{1}{m_c}-\frac{1}{m_b}\right)\frac{M_B-M_D}{M_B+M_D}-\mathcal{O}\left(\frac{1}{m_Q^2}\right)\;; \tag{29}$$

it is positive, explicitly suppressed by two powers of β in accord with the theorem in the previous section, and is small numerically. One finds that linear μ/m_Q dependence of one-loop $\xi_V(\mu)$ cancels against the one-loop μ-dependence of $\frac{\overline{\Lambda}}{2}-\overline{\Sigma}$, as expected.

Although $\frac{\overline{\Lambda}}{2}-\overline{\Sigma}$ normalized at 0.8 to 1 GeV has not been directly measured, a good estimate for it would be the actual upper bound [16]

$$\frac{\overline{\Lambda}}{2}-\overline{\Sigma}=\frac{1}{3}\frac{1}{2M_B}\langle B|\bar{b}(\vec{\sigma}\vec{\pi})(-\pi_0^{-1})(\vec{\sigma}\vec{\pi})b|B\rangle\;\le\;\sqrt{\left(\varrho^2-\tfrac{3}{4}\right)\frac{\mu_\pi^2-\mu_G^2}{3}}\;. \tag{30}$$

[5]A connection between the $1/m_Q$ correction and inclusive transitions was first noted in Ref. [19].

The IW slope ϱ^2 has not been well determined experimentally yet. An alternative estimate would use the scale of an average excitation energy $\tilde{\varepsilon}$ of the P-wave $\frac{1}{2}$-states saturating the sum rules:

$$\frac{\overline{\Lambda}}{2} - \overline{\Sigma} \;\approx\; \frac{\mu_\pi^2 - \mu_G^2}{3\tilde{\varepsilon}} \tag{31}$$

with $\tilde{\varepsilon} \approx 500$ to $700\,\mathrm{MeV}$. Depending on the precise value of $\mu_\pi^2(1\,\mathrm{GeV})$ the $1/m$ correction to $\mathcal{F}_+$ emerges at 1% level.

Since the expansion in $1/m_c$ is involved, higher-order corrections *a priori* could be significant. This is illustrated by the perturbative contribution where linear in μ approximation works well only for $\mu \lesssim 400\,\mathrm{MeV}$, while already at $\mu \simeq 800\,\mathrm{MeV}$ second- and third-order terms are of the same size. The BPS expansion ensures, however that the overall suppression is carried on to all higher-order nonperturbative effects as well.

The second-order power correction to $\mathcal{F}_+$ has three pieces, each manifestly of the second order in BPS. It also follows directly from Eq. (25), now explicitly expanding in $1/m_c$:

$$\delta_{1/m^2}\mathcal{F}_+ = \left(\frac{1}{m_c} - \frac{1}{m_b}\right)^2 \{V_1 + V_2 + V_3\} \tag{32}$$

The first one, V_1 is the same as in zero-recoil $\langle D|\bar{c}\gamma_0|B\rangle$ and is well understood [14]. It is negative consisting of local correction and nonlocal "overlap deficit":

$$V_1 = -\frac{\mu_\pi^2 - \mu_G^2}{8}\cdot\left(1 + \chi_{\text{n-l}}^{(V)}\right), \qquad (\mu_\pi^2 - \mu_G^2)\,\chi_{\text{n-l}}^{(V)} = \frac{1}{2M_B}\langle B|\bar{b}(\vec{\sigma}\vec{\pi})^2\pi_0^{-2}(\vec{\sigma}\vec{\pi})^2 b|B\rangle$$

$$= \int i|x_0|\,\mathrm{d}^4x\,\frac{1}{4M_B}\langle B|\,iT\{\bar{b}(\vec{\sigma}\vec{\pi})^2 b(x),\,\bar{b}(\vec{\sigma}\vec{\pi})^2 b(0)\}\,|B\rangle' > 0\,. \tag{33}$$

The second piece is local and positive, $V_2 = \frac{\mu_\pi^2 - \mu_G^2}{6}$. The last piece can be viewed as the $1/m_Q$ correction to the value of $\frac{\overline{\Lambda}}{2} - \overline{\Sigma}$ in a finite-mass meson and is described by the expectation value of the nonlocal correlator

$$-6V_3 = \frac{1}{2M_B}\langle B|\bar{b}(\vec{\sigma}\vec{\pi})^2\pi_0^{-1}(\vec{\sigma}\vec{\pi})\pi_0^{-1}(\vec{\sigma}\vec{\pi})b|B\rangle'\,. \tag{34}$$

It is plausible that the correlator is likewise positive decreasing the effective value of $\frac{\overline{\Lambda}}{2} - \overline{\Sigma}$, although strictly speaking the sign is not fixed. Its scale can be roughly estimated as that for the correlator $\bar{Q}(\vec{\sigma}\vec{\pi})\pi_0^{-1}(\vec{\sigma}\vec{\pi})^2\pi_0^{-1}(\vec{\sigma}\vec{\pi})Q$ and taking for the latter $\frac{\mu_\pi^2}{\tilde{\varepsilon}}\left(\frac{\overline{\Lambda}}{2} - \overline{\Sigma}\right)$ as an educated dimensional guess. Since local pieces nearly cancel, $\delta_{1/m^2}\mathcal{F}_+$ is dominated by two nonlocal correlators.

Collecting all terms through the second order in $1/m_Q$ we arrive at the estimate

$$\delta_{\mathrm{power}}\,\mathcal{F}_+ \lesssim 0.01 \;\text{ at } \mu_\pi^2 \lesssim 0.43\,\mathrm{GeV}^2\,, \tag{35}$$

where following Refs. [14, 20] we assume $\chi_{\text{n-l}}^{(V)} = 0.5 \pm 0.5$. The literal prediction depends moderately on the actual kinetic expectation value,

$$\mathcal{F}_+ \simeq 1.04 + 0.13\,\frac{\mu_\pi^2(1\,\mathrm{GeV}) - 0.43\,\mathrm{GeV}^2}{1\,\mathrm{GeV}^2}\;; \tag{36}$$

the uncertainty, however would soon go out of control at this level of precision for $\mu_\pi^2(1\,\mathrm{GeV})$ exceeding $0.45\,\mathrm{GeV}^2$.

The above estimates suggest that the nonperturbative corrections to $\mathcal{F}_+$ are really tiny, and it can be accurately evaluated. It should be appreciated, however that practical implementations of the heavy quark expansion leave out possible exponential terms like

$$\delta_{\rm exp}\,\mathcal{F}_+ \propto \left(e^{-\frac{m_c}{\mu_{\rm hadr}}} - e^{-\frac{m_b}{\mu_{\rm hadr}}}\right)^2 \tag{37}$$

which are routinely ignored, but may be essential at a percent level when relying on heavy charm expansion [21]. At the moment we cannot say much about them; yet they probably put the actual limit on the accuracy of the theoretical predictions for $\mathcal{F}_+$ if $\mu_\pi^2(1\,\mathrm{GeV})$ is confirmed to be below $0.45\,\mathrm{GeV}^2$.

4 Conclusions

We have analyzed the structure of the pure nonperturbative corrections to B and D mesons near the 'BPS' regime which would apply if nonperturbative physics is largely confined below 1 GeV scale and yield μ_π^2 close to μ_G^2. A number of relations are shown to hold extending the heavy flavor (but not spin!) symmetry to all orders in $1/m_Q$ in the BPS limit, valid however only for the ground state pseudoscalar heavy flavor mesons. Some of the important BPS relations get corrections only to the second order in the BPS expansion regardless of the order in $1/m_Q$, among them are two $B\to D$ zero recoil formfactors. This is the analogue of the Ademollo-Gatto theorem for the BPS expansion.

The practical utility of such an expansion strongly depends on the actual size of $\mu_\pi^2(1\,\mathrm{GeV})$. If – as has been reported by most experimental analyses – it centers at or below 0.4, or even up to $0.45\,\mathrm{GeV}^2$, it is rather powerful. At larger values of μ_π^2 the predictability weakens; in this case, however any application relying on the heavy quark expansion for charm for non-BPS–protected relations would become questionable.

Dynamic heavy quark expansion has enlightened us that the scale of nonperturbative effects for heavy quarks in actual QCD is quite significant, $\mu^{\rm np} \gtrsim \sqrt{\mu_\pi^2} \simeq 700\,\mathrm{MeV}$ at least. This contrasted early ideas that this scale is like a 'constituent' light quark mass $m_{\rm const}\sim 250\,\mathrm{MeV}$ inherited from naive nonrelativistic quark models. Yet it turns out that the latter small scale can sometimes re-emerge in theory – where the 'BPS'-protected properties are considered, with $\sqrt{\mu_\pi^2-\mu_G^2}\lesssim 300\,\mathrm{MeV}$. A better theoretical understanding of the dynamic origin of such a hierarchy is highly desirable.

An interesting place to test the BPS predictions is the ratio $f_-(0)/f_+(0)$ fixed in the heavy quark limit,

$$\frac{f_-(0)}{f_+(0)} = -\frac{M_B-M_D}{M_B+M_D}\;. \tag{38}$$

Since both $f_+(0)$ and $f_-(0)$ undergo power corrections only to the second order in the BPS expansion, they must be small and accurately evaluable, Sect. 3, thereby offering a probe for possible exponential effects. The suppressed formfactor f_- can be measured in the

$B \to D\,\tau\nu_\tau$ decays in future high-statistics experiments. This mode also probes possible effect of charge Higgs exchanges, which requires a precision understanding of the SM amplitude. The BPS expansion provides support for such a treatment; the electroweak corrections are to be properly incorporated at this level.

The heavy quark expansion together with heavy quark sum rules allows an accurate prediction for the $B \to D$ amplitudes near zero recoil in terms of measured observables. Our estimate (the electroweak effects, in particular the factor of 1.007 from the universal short-distance renormalization are not included here) depends on $\mu_\pi^2(1\,\mathrm{GeV})$, but for moderate values the power corrections appear at a percent level:

$$\mathcal{F}_+ = 1.04 \pm 0.01_{\rm power} \pm 0.01_{\rm pert} + \delta_{\rm exp}\;, \tag{39}$$

with perturbative corrections from momenta above 1 GeV contributing the dominant piece of 3%. It can be further refined. The corrections are significantly smaller and more definite compared to the 'gold plated' $B \to D^*$ decay mode.

This assessment differs from the existing estimates [22] although, in principle is compatible with the predictions within their respective large error bars. The rationale is readily seen equating and counting together orders in conventional $1/m_c$ and in the 'BPS' expansion: Eq. (29) gives power corrections through the **third** order, with estimates included for the fourth-order effects. In contrast, no BPS-backup exists for $B \to D^*$, the corrections are significant in the BPS regime [8], and they are uncertain already to the leading order $1/m_c^2$.

Acknowledgments: I am grateful to I. Bigi, M. Shifman and in particular to A. Vainshtein for interest, illuminating discussions and for sharing a nontrivial alternative perspective. This work was supported in part by the NSF under grant number PHY-0087419.

References

[1] K. Wilson, *Phys. Rev.* **179** (1969) 1499.

[2] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl. Phys.* **B147** 385 (1979); V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl. Phys.* **B249** 445 (1985).

[3] M. Voloshin and M. Shifman, *Yad. Phys.* **45** (1987) 463 [*Sov. J. Nucl. Phys.* **45** (1987) 292]; *ZhETF* **91** (1986) 1180 [*JETP* **64** (1986) 698].

[4] I. Bigi, N. Uraltsev and A. Vainshtein, *Phys. Lett.* **B293** (1992) 430

[5] N. Uraltsev, *Phys. Lett.* **B501** (2001) 86; *J. Phys. G* **27** (2001) 1081.

[6] M. Voloshin and M. Shifman, *Yad. Fiz.* **47** (1988) 801 [*Sov. J. Nucl. Phys.* **47** (1988) 511].

[7] A. Le Yaouanc, L. Oliver and J.-C. Raynal, *Phys. Lett.* **B557** (2003)207; *Phys. Rev.* **D67** (2003) 114009.

[8] N. Uraltsev, *Phys. Lett.* **B545** (2002) 337.

[9] M. Battaglia *et al.*, *Phys. Lett.* **B 556** (2003) 41.

[10] N. Uraltsev, Proc. of the 31st International Conference on High Energy Physics, Amsterdam, The Netherlands, 25-31 July 2002 (North-Holland – Elsevier, The Netherlands, 2003), S. Bentvelsen, P. de Jong, J. Koch and E. Laenen Eds., p. 554; hep-ph/0210044.

[11] I.I. Bigi and N. Uraltsev, hep-ph/0308165, *Phys. Lett.* **B** (to appear).

[12] N. Uraltsev, Proc. of the 2nd Workshop on the CKM Unitarity Triangle, IPPP Durham, April 2003, P. Ball, J.M. Flynn, P. Kluit and A. Stocchi, eds. (Electronic Proceedings Archive eConf C0304052, 2003); hep-ph/0306290;
hep-ph/0308165, talk at Int. Conference "Flavor Physics & CP Violation 2003" , June 3-6 2003, Paris. To appear in the Proceedings.

[13] K.C. Bowler *et al.* [UKQCD Collaboration], *Nucl. Phys.* **B637** (2002) 293.

[14] I.I. Bigi, M. Shifman, N.G. Uraltsev and A. Vainshtein, *Phys. Rev.* **D52** (1995) 196.

[15] I. Bigi, M. Shifman and N.G. Uraltsev, *Ann. Rev. Nucl. Part. Sci.* **47** (1997) 591.

[16] N. Uraltsev, in *Boris Ioffe Festschrift* "`At the Frontier of Particle Physics -- Handbook of QCD`", Ed. M. Shifman (World Scientific, Singapore, 2001), Vol. 3, p. 1577; hep-ph/0010328.

[17] Ref. [14], Sect. II.

[18] Ref. [14], Sect. VI.

[19] A. Le Yaouanc *et al.*, *Phys. Lett.* **B480** (2000) 119.

[20] M. Shifman, N.G. Uraltsev and A. Vainshtein, *Phys. Rev.* **D51** (1995) 2217.

[21] I. Bigi and N. Uraltsev, *Int. J. Mod. Phys.* **A16** (2001) 5201.

[22] For a dedicated review, see: M. Battaglia, A.J. Buras, P. Gambino and A. Stocchi, eds. *Proceedings of the* First Workshop on the CKM Unitarity Triangle, CERN, February 2002;